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Numerical Treatment for ODE (Fifth order)

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ABSTRACT: In the present article, a numerical technique with seven step block method is proposed to find approximate solution of fifth order differential equation with initial value by block integrator method without tackling the reduction method. The convergence, stability, consistency and order of the method are derived. The accuracy of the method over other method (ODE-45) is established through three numeral examples and absolute errors are determined.

Keywords: Block method, Collocation, Differential equation, Interpolation, Power series, Taylor series.

I. INTRODUCTION

The mathematical model of many problems in quantum physics are IVPs of the from

$$f(x,y,y',y'',y''',y^{iv},y''',y'',y',y,x) = y^{v},\ y_0 = \gamma_0,\ y'_0 = \gamma_1,\ y''_0 = \gamma_2,\ y'''_0 = \gamma_3,\ y_0^{iv} = \gamma_4 \quad (1)$$

Solution of initial value problems (IVPs) up to fourth order have been approached using different multistep methods and block methods by authors [1, 4-6] and [10-13] for the methods using predictor-corrector method starting values are obtained by Runge-Kutta method [7-9]. The direct approach with linear multistep method [15,18], efficient zero-stable numerical method using predictor-corrector mode [14], a five-step block method [17-19], six-step and seven step block method [16, 18] are applied to obtain approximate solution of Eqn. (1). In the present article, an efficient numerical scheme is employed for fifth order differential equation via block method.

To overcome the drawbacks of complex process, waste of time and large computer memory required, researchers have worked on developing block methods for better approximation [2, 3]. In this method values are calculated simultaneously at different grid points. In the present article a seven point block method is derived, which is zero stable, consistent and convergent.

The present work is organized as follows: In Section II the block method is constructed. In Section III an analysis and properties of the proposed method are analyzed. Numerical verification and brief comparison with ODE-45 are determined in Section IV. The conclusions are drawn in Section V.

II. DERIVATION OF THE METHOD

The Eqn. (1) is expanded by the power series,

$$y(x) \approx \sum_{i=0}^{12} a_i x^i$$

or equivalently

$$y(x) \approx \sum_{i=0}^{12} a_i \left(\frac{x-x_n}{h}\right)^i \quad (2)$$

and Eqn. (2) is differentiated up to fifth order, yields.

$$y^{v}(x) = \sum_{i=5}^{12} i\,(i-1)(i-2)(i-3)(i-4)a_i \left(\frac{x-x_n}{h}\right)^{i-5} \quad (3)$$

The system of equations are obtained interpolating Eqn. (2) at the grid points $x_n, x_{n+1}, x_{n+2}, x_{n+3}, x_{n+4}$ and collating Eqn. (3) at x_{n+i}, $i = 0(1)7$. The values of the coefficients $a_i, i = 0(1)12$ are obtained by matrix inversion method.

The calculated values when substituted to Eqn. (2), we obtain LMM of the form

$$y(x) = \sum_{i=0}^{4} \alpha_i y_{n+i} + \sum_{j=0}^{7} \beta_j f_{n+j} \quad (4)$$

Putting $t = \frac{x-x_n}{h}$ in Eqn. (4), α_i, β_j are calculated as follows:

$$\alpha_0 = \frac{1}{24}t^4 - \frac{5}{12}t^3 + \frac{35}{24}t^2 - \frac{25}{12}t + 1,$$

$$\alpha_1 = \frac{3t^3}{2} - \frac{13t^2}{3} + 4t,$$

$$\alpha_2 = \frac{1}{4}t^4 - 2t^3 + \frac{19t^2}{4} - 3t,$$

$$\alpha_3 = \frac{1}{24}t^4 - \frac{1}{4}t^3 + \frac{11}{24}t^2 - \frac{1}{4},$$

$$\alpha_4 = \frac{11}{24}t^2 - \frac{1}{4}t$$

$$\beta_0 = \frac{t^{12}}{861298532} + \frac{5t^{11}}{731934} + \frac{t^{10}}{1262212998} - \frac{10t^9}{98667} - \frac{10t^8}{98667} + \frac{30t^7}{40627} - \frac{39t^6}{12307} + \frac{t^5}{120} - \frac{127t^4}{9654} + \frac{109t^3}{9509} - \frac{35t^2}{7899} + \frac{47t}{149209},$$

$$\beta_1 = -\frac{t^{12}}{160761110} - \frac{7t^{11}}{220074} - \frac{t^{10}}{196819654} + \frac{5t^9}{11816} + \frac{5t^8}{11816} - \frac{17t^7}{6721} + \frac{3668t^6}{52661} - \frac{177t^4}{3145} + \frac{397t^3}{2522} - \frac{167t^2}{927} + \frac{212t}{2857},$$

$$\beta_2 = \frac{t^{12}}{74825575} + \frac{9t^{11}}{153311} + \frac{t^{10}}{71681232} - \frac{t^9}{15680} - \frac{11t^8}{15680} + \frac{79t^7}{22466} - \frac{85t^6}{12192} - \frac{29t^4}{5666} + \frac{326t^3}{3043} - \frac{439t^2}{2027} + \frac{227t}{1913},$$

$$\beta_3 = -\frac{t^{12}}{70660188} - \frac{9t^{11}}{170630} - \frac{t^{10}}{47013602} + \frac{13t^9}{22759} + \frac{13t^8}{22759} - \frac{64t^7}{25331} + \frac{43t^6}{9351} - \frac{223t^4}{18860} + \frac{70t^3}{3887} - \frac{57t^2}{4070} + \frac{337t}{64442},$$

$$\beta_4 = \frac{t^{12}}{148440642} + \frac{4t^{11}}{190497} + \frac{t^{10}}{51382122} - \frac{31t^9}{147534} - \frac{31t^8}{147534} + \frac{71t^7}{81117} - \frac{31t^6}{20206} + \frac{43t^4}{12980} - \frac{82t^3}{31991} + \frac{40t^2}{24517} + \frac{1373t}{793593},$$

$$\beta_5 = -\frac{t^{12}}{60826645440} + \frac{t^{11}}{10461585} - \frac{t^{10}}{93497259} - \frac{t^9}{2006005} - \frac{t^8}{2006005} + \frac{t^7}{668668} - \frac{t^6}{457151} + \frac{4t^4}{921709} - \frac{t^3}{239375} - \frac{t^2}{2081027} + \frac{t}{702618},$$

$$\beta_6 = -\frac{t^{12}}{836728085} - \frac{t^{11}}{370889} + \frac{t^{10}}{305588410} + \frac{3t^9}{121303} + \frac{3t^8}{121303} - \frac{23t^7}{236112} + \frac{5t^6}{30293} - \frac{4t^4}{11409} + \frac{17t^3}{60960} + \frac{22t^2}{135257} - \frac{7t}{38754},$$

$$\beta_7 = \frac{t^{12}}{3104908980} + \frac{t^{11}}{1611161} - \frac{t^{10}}{2322471917} - \frac{3t^9}{536918} - \frac{3t^8}{536918} + \frac{9t^7}{414314} - \frac{2t^6}{54777} + \frac{3t^4}{38806} - \frac{20t^3}{321039} - \frac{7t^2}{203708} + \frac{22t}{562453}.$$

Evaluating Eqn. (4) with it's derivative upto 4th order at select grid points, we obtain a discrete block series of the form

$$A^0 Y^i_{n+k} = A_1 Y_{n-k} + hA_2 Y'_{n-k} + h^2 A_3 Y''_{n-k} + h^3 A_4 Y'''_{n-k} + h^4 A_5 Y^{iv}_{n-k} + h^5 B^i F_{n+k} + h^5 D^i F_{n-k} \quad (5)$$

Where,

$$Y_{n+k} = [y_{n+1}, y_{n+2}, y_{n+3}, y_{n+4}, y_{n+5}, y_{n+6}, y_{n+7}]^T$$

$$Y^{(i)}_{n+k} = \left[y^{(i)}_{n+1}, \dots\dots\dots, y^{(i)}_{n+7}\right]$$

$$Y^{(i)}_{n-k} = \left[y^{(i)}_{n-k+1}, y^{(i)}_{n-k+2}, \dots\dots\dots, y^{(i)}_{n}\right]$$

$$F_{n+k} = [f_{n+1}, f_{n+2}, \dots\dots\dots\, f_{n+k}]^T$$

$$F_{n-k} = [f_{n-k+1}, f_{n-k+2}, \dots\dots\dots\, f_{n}]^T$$

$$A^0 = \begin{bmatrix} 1&0&0&0&0&0&0\\ 0&1&0&0&0&0&0\\ 0&0&1&0&0&0&0\\ 0&0&0&1&0&0&0\\ 0&0&0&0&1&0&0\\ 0&0&0&0&0&1&0\\ 0&0&0&0&0&0&1 \end{bmatrix}, \quad A^1 = \begin{bmatrix} 0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0&0&0&0&0&0&1\\ 0&0&0&0&0&0&2\\ 0&0&0&0&0&0&3\\ 0&0&0&0&0&0&4\\ 0&0&0&0&0&0&5\\ 0&0&0&0&0&0&6\\ 0&0&0&0&0&0&7 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0&0&0&0&0&0&\frac{1}{2}\\ 0&0&0&0&0&0&2\\ 0&0&0&0&0&0&\frac{9}{2}\\ 0&0&0&0&0&0&8\\ 0&0&0&0&0&0&\frac{25}{2}\\ 0&0&0&0&0&0&18\\ 0&0&0&0&0&0&\frac{49}{2} \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0&0&0&0&0&0&\frac{1}{6}\\ 0&0&0&0&0&0&\frac{4}{3}\\ 0&0&0&0&0&0&\frac{9}{2}\\ 0&0&0&0&0&0&\frac{32}{3}\\ 0&0&0&0&0&0&\frac{125}{6}\\ 0&0&0&0&0&0&36\\ 0&0&0&0&0&0&\frac{343}{6} \end{bmatrix}, \quad A_5 = \begin{bmatrix} 0&0&0&0&0&0&\frac{1}{24}\\ 0&0&0&0&0&0&\frac{2}{3}\\ 0&0&0&0&0&0&\frac{27}{8}\\ 0&0&0&0&0&0&\frac{32}{3}\\ 0&0&0&0&0&0&\frac{625}{24}\\ 0&0&0&0&0&0&54\\ 0&0&0&0&0&0&\frac{2401}{24} \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0&0&0&0&0&0&\frac{9917}{1774080}\\ 0&0&0&0&0&0&\frac{9623}{74844}\\ 0&0&0&0&0&0&\frac{737073}{985600}\\ 0&0&0&0&0&0&\frac{35968}{14175}\\ 0&0&0&0&0&0&\frac{61946875}{9580032}\\ 0&0&0&0&0&0&\frac{21249}{1540}\\ 0&0&0&0&0&0&\frac{59513587}{2280960} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0&0&0&0&0&0&\frac{443243}{17107200}\\ 0&0&0&0&0&0&\frac{132526}{467775}\\ 0&0&0&0&0&0&\frac{526077}{492800}\\ 0&0&0&0&0&0&\frac{1250432}{467775}\\ 0&0&0&0&0&0&\frac{25842625}{4790016}\\ 0&0&0&0&0&0&\frac{18342}{1925}\\ 0&0&0&0&0&0&\frac{262892693}{17107200} \end{bmatrix}$$

$$B_2=\begin{bmatrix}0&0&0&0&0&0&\frac{12437}{134400}\\0&0&0&0&0&0&\frac{6688}{14175}\\0&0&0&0&0&0&\frac{51327}{44800}\\0&0&0&0&0&0&\frac{9992}{4725}\\0&0&0&0&0&0&\frac{490375}{145152}\\0&0&0&0&0&0&\frac{864}{175}\\0&0&0&0&0&0&\frac{1173403}{172800}\end{bmatrix},\quad B_3=\begin{bmatrix}0&0&0&0&0&0&\frac{416173}{1814400}\\0&0&0&0&0&0&\frac{14939}{28350}\\0&0&0&0&0&0&\frac{18399}{22400}\\0&0&0&0&0&0&\frac{15824}{14175}\\0&0&0&0&0&0&\frac{102425}{72576}\\0&0&0&0&0&0&\frac{597}{350}\\0&0&0&0&0&0&\frac{519253}{259200}\end{bmatrix},\quad B_4=\begin{bmatrix}0&0&0&0&0&0&\frac{5257}{17280}\\0&0&0&0&0&0&\frac{41}{140}\\0&0&0&0&0&0&\frac{265}{896}\\0&0&0&0&0&0&\frac{278}{945}\\0&0&0&0&0&0&\frac{265}{896}\\0&0&0&0&0&0&\frac{41}{140}\\0&0&0&0&0&0&\frac{5257}{17280}\end{bmatrix}$$

$$D_0=\begin{bmatrix}\frac{425111}{68428800}&-\frac{126827}{15966720}&\frac{269923}{31933440}&-\frac{297547}{4790016}&\frac{94093}{31933440}&-\frac{64837}{7983360}&\frac{1895}{19160064}\\ \frac{4868}{18711}&-\frac{8159}{29700}&\frac{2438}{8505}&-\frac{78119}{374220}&\frac{1024}{10395}&-\frac{10127}{374220}&\frac{1538}{467775}\\ \frac{3961467}{1971200}&-\frac{1690551}{985600}&\frac{20817}{11264}&-\frac{265761}{197120}&\frac{1254609}{1971200}&-\frac{172341}{985600}&\frac{3807}{179200}\\ \frac{3735296}{467775}&-\frac{855104}{155925}&\frac{204032}{31185}&-\frac{63488}{13365}&\frac{349952}{155925}&-\frac{10688}{17325}&\frac{35072}{467775}\\ \frac{430015625}{19160064}&-\frac{13328125}{1064448}&\frac{335703125}{19160064}&-\frac{10703125}{870912}&\frac{1773125}{304128}&-\frac{15359375}{9580032}&\frac{3734375}{19160064}\\ \frac{98496}{1925}&-\frac{36207}{1540}&\frac{15282}{385}&-\frac{39933}{1540}&\frac{972}{77}&-\frac{3807}{1100}&\frac{162}{385}\\ \frac{126237377}{1244160}&-\frac{4434190811}{11404800}&\frac{21296119}{1520640}&\frac{323417101}{6842880}&\frac{22470959}{912384}&-\frac{14941423}{2280960}&\frac{54841241}{68428800}\end{bmatrix}$$

$$D_1=\begin{bmatrix}\frac{36943}{1069200}&-\frac{63241}{147800}&\frac{270751}{5987520}&\frac{793561}{23950080}&\frac{8699}{554400}&-\frac{517351}{119750400}&\frac{15739}{29937600}\\ \frac{315461}{467775}&-\frac{974}{1485}&\frac{64067}{93555}&-\frac{9322}{18711}&\frac{12211}{51975}&-\frac{30176}{467775}&\frac{733}{93555}\\ \frac{404109}{123200}&-\frac{1195317}{492800}&\frac{963}{352}&-\frac{197073}{98560}&\frac{116397}{123200}&-\frac{127971}{127971}&\frac{243}{7700}\\ \frac{877568}{93555}&-\frac{91264}{17325}&\frac{134144}{18711}&-\frac{68512}{13365}&\frac{41984}{17325}&-\frac{62336}{93555}&\frac{37888}{467775}\\ \frac{12274375}{598752}&-\frac{4748125}{532224}&\frac{18690625}{1197504}&-\frac{49290625}{4790016}&\frac{5875}{1188}&-\frac{6516875}{4790016}&\frac{198125}{1197504}\\ \frac{73467}{1925}&-\frac{25272}{1925}&\frac{11493}{385}&-\frac{6642}{385}&\frac{17253}{1925}&-\frac{666}{275}&\frac{81}{275}\\ \frac{273298627}{273298627}&-\frac{2235331}{126720}&\frac{5529503}{106920}&-\frac{17529701}{684288}&\frac{823543}{52800}&-\frac{64354003}{17107200}&\frac{40817}{85536}\end{bmatrix}$$

$$D_2=\begin{bmatrix}\frac{40187}{259200}&-\frac{220919}{1209600}&\frac{23141}{120960}&-\frac{20267}{145152}&\frac{39901}{604800}&-\frac{21941}{1209600}&\frac{4001}{1814400}\\ \frac{19381}{14175}&-\frac{521}{450}&\frac{3473}{2835}&-\frac{506}{567}&\frac{221}{525}&-\frac{3277}{28350}&\frac{199}{14175}\\ \frac{91827}{22400}&-\frac{105381}{44800}&\frac{387}{128}&-\frac{19791}{8960}&\frac{23409}{22400}&-\frac{12879}{44800}&\frac{783}{22400}\\ \frac{118432}{14175}&-\frac{15536}{4725}&\frac{5792}{945}&-\frac{1672}{405}&\frac{9248}{4725}&-\frac{848}{1575}&\frac{928}{14175}\\ \frac{1026125}{72576}&-\frac{7125}{1792}&\frac{801875}{72576}&-\frac{891875}{145152}&\frac{3625}{1152}&-\frac{125375}{145152}&\frac{7625}{72576}\\ \frac{3753}{175}&-\frac{1539}{350}&\frac{621}{35}&-\frac{54}{7}&\frac{891}{175}&-\frac{63}{50}&\frac{27}{175}\\ \frac{7848869}{259200}&-\frac{789929}{172800}&\frac{16807}{640}&-\frac{184877}{20736}&\frac{722701}{86400}&-\frac{218491}{172800}&\frac{57281}{259200}\end{bmatrix}$$

$$D_3 = \begin{bmatrix} \frac{33953}{64800} & -\frac{341699}{604800} & \frac{105943}{181440} & -\frac{153761}{362880} & \frac{943}{4725} & -\frac{99359}{1814400} & \frac{6031}{907200} \\ \frac{27821}{14175} & -\frac{14175}{675} & \frac{799}{567} & -\frac{5881}{5670} & \frac{2321}{4725} & -\frac{1916}{14175} & \frac{233}{14175} \\ \frac{39141}{11200} & -\frac{24111}{22400} & \frac{369}{160} & -\frac{7299}{4480} & \frac{8613}{11200} & -\frac{4737}{22400} & \frac{9}{350} \\ \frac{71152}{14175} & -\frac{(3832}{4725} & \frac{11344}{2835} & -\frac{856}{405} & \frac{4912}{4725} & -\frac{4072}{14175} & \frac{496}{14175} \\ \frac{59375}{9072} & -\frac{13375}{24192} & \frac{210625}{36288} & -\frac{130625}{72576} & \frac{1225}{864} & -\frac{26875}{72576} & \frac{1625}{36288} \\ \frac{1413}{175} & -\frac{54}{175} & \frac{267}{35} & -\frac{99}{70} & \frac{459}{175} & -\frac{9}{25} & \frac{9}{175} \\ \frac{1241317}{129600} & \frac{2401}{86400} & \frac{12005}{1296} & -\frac{40817}{51840} & \frac{160867}{43200} & \frac{146461}{259200} & \frac{8183}{64800} \end{bmatrix}$$

$$D_4 = \begin{bmatrix} \frac{139849}{120960} & -\frac{4511}{4480} & \frac{123133}{120960} & -\frac{88547}{120960} & \frac{1537}{4480} & -\frac{11351}{120960} & \frac{275}{24192} \\ \frac{1466}{945} & -\frac{71}{420} & \frac{68}{105} & -\frac{1927}{3780} & \frac{26}{105} & -\frac{29}{420} & \frac{8}{945} \\ \frac{1359}{896} & \frac{1377}{4480} & \frac{5927}{4480} & -\frac{3033}{4480} & \frac{1377}{4480} & -\frac{373}{4480} & \frac{9}{896} \\ \frac{1448}{945} & \frac{8}{35} & \frac{1784}{945} & -\frac{106}{945} & \frac{8}{35} & -\frac{64}{945} & \frac{8}{945} \\ \frac{36725}{24192} & \frac{775}{2688} & \frac{4625}{2688} & \frac{13625}{24192} & \frac{1895}{2688} & -\frac{275}{2688} & \frac{275}{24192} \\ \frac{54}{35} & \frac{27}{140} & \frac{68}{35} & \frac{27}{140} & \frac{54}{35} & -\frac{41}{140} & 0 \\ \frac{25039}{17280} & \frac{343}{640} & \frac{20923}{17280} & \frac{20923}{17280} & \frac{343}{640} & -\frac{25039}{17280} & \frac{5257}{17280} \end{bmatrix}$$

III. ANALYSIS AND CHARACHTERISTICS

A. Order of the method

Eqn. (5) can expressed with linear operator

$$L = Y_{n+k}^{(i)} - \sum_{j=i}^{4-i} h^j E_j Y_{n-k}^{(i)} - h^{5-i}(B_i F_{n-k} + D_i Y_{n+k}) \quad (6)$$

for the above calculated values of A^0, E_j, B_i and D_i. Expanding Eqn. (6) by Taylor's series

$$[L(y(x),h)] = \gamma_0 y(x) + \gamma_1 h y'^{(x)} + \gamma_2 h^2 y''^{(x)} + \cdots\cdots\cdots + \gamma_{p+2} h^{p+2} y^{p+2}(x),$$

Using Lambert (1973) [8], $\gamma_0 = \gamma_1 = \cdots\cdots = \gamma_p = \gamma_{p+1} = \gamma_{p+2} = 0$ and $\gamma_{p+3} \neq 0$. $p = 10$ and

$$\gamma_{13} = \begin{bmatrix} -\frac{9}{107359}, -\frac{76}{27629}, -\frac{79}{4449}, -\frac{574}{9174}, -\frac{519}{3178}, \\ -\frac{1090}{3089}, -\frac{775}{1152} \end{bmatrix}^T$$

where p and γ_{p+2} are error and error constant, respectively

Proposition-1: A method is consistent if it has order greater than one. In our case the order of the method is 10. Hence the present method is consistent.

B. Stability Analysis

Proposition-2: In the present work the first characteristic equation is given

$$\rho(\lambda) = \left| \lambda \begin{bmatrix} \lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\lambda^6(\lambda - 1) = 0,$$

$\lambda = 0$ is of multiplicity six which is less than seven and $\lambda = 1$ are the solutions of Eqn. (6). So the method is zero stable.

C. Convergence

Proposition-3: The proposed method is zero stable, so it is convergent.

IV. NUMERICAL EXAMPLES

1. $y^{v} = -(\cos x + \sin x),\ y_0 = y_0' = 1, y_0'' = -2, y_0''' = 1, y_0^{iv} = 2; 0 \le x \le 1$ and $h = 0.1$

The exact solution is

$y = 2x - \frac{1}{2}x^2 + \frac{1}{24}x^4 + \cos x - \sin x.$

2. $y^{v} = 5y''' - 4y', y_0 = 3, y_0' = -5, y_0'' = 11,$ $y_0''' = -23, y_0^{iv} = 47, 0 \le x \le 1$ and $h = 0.1$

The analytical solution is

$y = 1 - e^{-x} + 3e^{-2x}$

3. $y^{v} = y^{iv} + y' - y,\ y_0 = y_0' = y_0''' = y_0^{iv} = 0, y_0'' = 1;$ $0 \le x \le 1$ and $h = 0.1$

Analytical solution is $y = \frac{1}{2}(\cosh x - \cos x)$

Table 1: Comparison of Approximate solutions with Analytical solutions of Test-1.

X	Analytical	ODE 45	Approx	Absolute Error by current Method
0.1	1.090174915297864	1.090174915016777	1.090174915297859	4 .884981308350689e-15
0.2	1.161463913712847	1.161463913179007	1.161463913712838	9.103828801926284e-15
0.3	1.215153782464266	1.215153781708555	1.2151537824642654	1.776356839400251e-15
0.4	1.252709318360901	1.252709317416430	1.252709318360978	7.704947790898586e-14
0.5	1.275761189952836	1.275761188854608	1.275761189952826	9.992007221626409e-15
0.6	1.286093141514643	1.286093140299185	1.286093141514631	1.199040866595169e-14
0.7	1.285628666713464	1.285628665418457	1.285628666713445	1.909583602355269e-14
0.8	1.276417285114309	1.276417283778195	1.276417285114310	1.110223024625157e-15
0.9	1.260620558643181	1.260620557304766	1.260620558643175	5.995204332975845e-15
1.0	1.240497987726910	1.240497986424961	1.240497987726971	6.106226635438361e-14

Table 2: Comparison of Approximate solutions and Analytical solutions of Test-2.

X	Analytical	ODE 45	Approx	Absolute Error by current Method
0.1	2.551354841197986	2.551354809036535	2.551354841197989	3.108624468950438e-15
0.2	2.192229385028936	2.192229408226443	2.192229385028912	2.398081733190338e-14
0.3	1.905616687600361	1.905616747101572	1.905616687607256	6.894929072132072e-12
0.4	1.677666846316025	1.677666928156961	1.677666846461389	1.453641651494309e-10
0.5	1.497107663801694	1.497107757889994	1.497107665483066	1.681371930573050e-09
0.6	1.354770999642580	1.354771098812708	1.354771011906766	1.226418588906597e-08
0.7	1.243205588033410	1.243205687317150	1.243205653777727	6.574431687944582e-08
0.8	1.156360589866745	1.156360685931992	1.156360870765622	2.808988770475196e-07
0.9	1.089327004924161	1.089327091384927	1.089328014461847	1.009537685892070e-06
1.0	1.038126408538396	1.038126510324552	1.038129573989230	3.165450833897410e-06

Table 3: Comparative study of Test-3.

X	Analytical	ODE 45	Approx	Absolute Error by current Method
0.1	0.005000001388889	0.005000001190181	0.005000001388878	1.099988156116893e-14
0.2	0.020000088888917	0.020000086817131	0.020000088888920	2.997602166487923e-15
0.3	0.045001012501627	0.045000971592646	0.045001012501613	1.399574900418088e-14
0.4	0.080005688917785	0.080005380737770	0.080005688917767	1.799949078673535e-14
0.5	0.125021701658004	0.125020245004635	0.125021701658015	1.099120794378905e-14
0.6	0.180064801666295	0.180059639621234	0.180064801666226	1.900036098045348e-14
0.7	0.245163409173227	0.245148396300883	0.245163409173212	1.498801083243961e-14
0.8	0.320364118478840	0.320326325669622	0.320364118478821	1.898481372109018e-14
0.9	0.405738208589055	0.405652994462851	0.405738208589043	1.199040866595169e-14
1.0	0.501389164473552	0.501213006791437	0.501389164473547	4.996003610813204e-15

V. CONCLUSION

In the present work a seven point block method is applied to solve fifth order IVPs directly using without reduction as well as predictors and correctors methods. The convergence, consistency and order of stability has been established. Implementation of the method in different types of problems and its comparison with the exact solution and ODE 45 has been reported in Table 1, 2 and 3, respectively. It is observed that our present method is a good agreement with the exact solution of fifth order differential equations of different approach in a nice manner as compared to other existing methods.

VI. FUTURE SCOPE

This work may be extended to obtain the approximate solution initial value problem of higher order ODE in physical sciences.

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